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Article

Uncovering interest-elastic money demand: Evidence from the Japanese money market with a low interest rate policy

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Abstract

This paper explores empirically the overnight call rate (interbank rates) below which extremely interest-elastic money demand emerges, using Japanese money market data for the sample period 1985 to 2001 in which a liquidity trap phenomenon was not necessarily self-evident. First, the presence of a cointegration relationship in money demand with possible breaks is confirmed by a test of Gregory and Hansen [9]. Then, exploiting the fact that short-run nominal interest rates had declined almost monotonically since the early 1990s, sophisticated tests of structural breaks constructed by Hansen [11] and Kuo [14] allow us to identify the rate of interest below which money demand is extremely interest-elastic. The paper presents the following empirical findings based on the cointegration estimation of the dynamic OLS and the fully modified OLS. First, money demand curves with extremely high interest-

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rate semi-elasticity have been observed since the Bank of Japan started to guide overnight call rates below 0.5% in 1995. Second, consistent with a possible implication from interest-elastic money demand, nominal prices did not respond to changes in nominal money supply in the short-run under the low interest rate policy. Small sample properties of our estimates are also discussed.

JEL classification: E31, E41, E52.

Keywords: money demand, zero interest rate policy, cointegration, structural break, dynamic OLS, fully modified OLS.

1. Introduction

A degree of interest-rate *semi-elasticity* of money demand has two important implications. The normative implication is that higher interest-elastic money demand involves larger deadweight losses due to a marginal increase in short-run interest rates. The positive implication is that if a central bank cannot make a firm commitment to permanent increases in the money supply, current nominal prices do not respond to changes in money supply under interest-elastic money demand. Such an empirical implication is also true of the case where market participants are myopic in the sense that they do not have long-run expectations about the money supply.

Almost infinitely elastic money demand, or a liquidity trap phenomenon, in the Japanese money market has been evident since the Bank of Japan adopted an aggressive quantity easing policy in March 2001. Under this monetary policy framework, high-powered money has expanded at zero overnight call rates (interbank rates) by gradually enlarging excess reserves at the BOJ checking account. As of July 2004, the size of the BOJ reserves reached above thirty trillion yen, while that of the required reserves amounted to only around five trillion yen. Given stable nominal prices, not only the money supply's nominal balances, but also its real balances had grown dramatically at zero short-run interest rates. Then, a horizontal money demand function has been observed in the neighborhood of zero interest rates; in

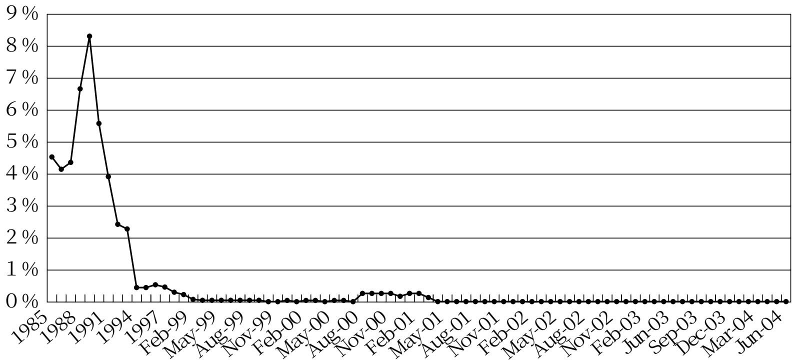

Figure 1: Overnight Call Rates (Overnight Interbank Rates) (monthly data from January 1999)

other words, a liquidity trap phenomenon had emerged.

Given the above fact, recent empirical works on the Japanese money markets including those of Miyao [18] and Fujiki and Watanabe [8], estimate money demand functions by regressing the logarithm of real money on the logarithm of nominal interest rates (the so-called log-log specification where interest-rate semi-elasticity is infinite at zero nominal interest rates).

Even before implementation of the quantity easing policy, however, money demand seemed stronger under an extremely low interest rate policy. From September 1995, the BOJ developed a low interest rate policy that had never been experienced before (see Figure 1). While the BOJ initially guided overnight call rates (interbank rates) below 0.5%, in September 1998 it lowered overnight rates to even less than 0.25%. In February 1999, the BOJ implemented its so-called zero interest rate policy, whereby the targeted overnight rate was set almost at zero. The zero interest rate policy was lifted in August 2000. The targeted rate, however, has since remained well below 0.5%. As mentioned above, the BOJ adopted its aggressive quantity policy in March 2001.

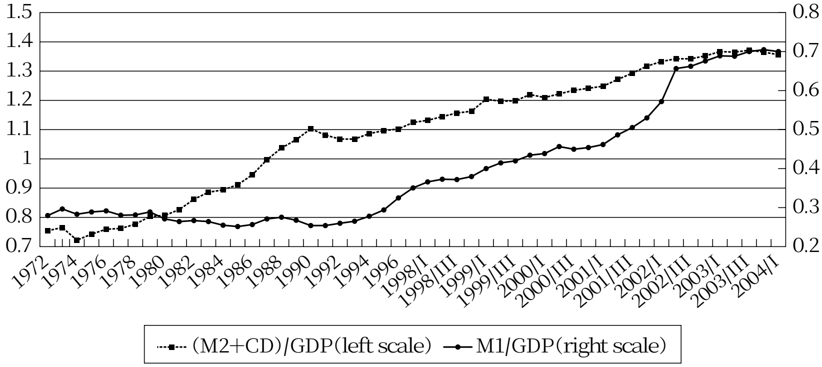

Figure 2: The Ratio of Monetary Aggregates Relative to Nominal GDP (seasonally adjusted for quarterly data from 1998/I)

According to Figure 2, the size of monetary aggregates represented by both M1 and M2 + CD relative to nominal GDP increased remarkably in the latter half of the 1990s when the low interest rate policy was implemented. In particular, the ratio based on M1, a monetary aggregate that mainly reflects the transaction motive, has expanded towards 40% and even higher since the middle of the 1990s, whereas it had been fairly stable between 25% and 30% until that time. Restating this phenomenon, the effect of aggressive expansion of money supply on nominal prices has been counteracted by fairly strong money demand or a rather slow velocity of money circulation since 1995.

Because a degree of interest-rate semi-elasticity itself has important normative and positive implications, this paper adopts a conventional semi-log specification in which the level, rather than the logarithm, of nominal interest rates is used as a regressor for money demand. Then, it explores empirically the rate of interest below which extremely strong money demand emerges, or extraordinarily interest-elastic money demand, using Japanese money market data for the sample period from January 1985 to March 2001. Our sample omits the period in which money demand is completely flat under the quantity easing policy, because interest-rate semi-elasticity is not identifiable for a flat curve under a semilog specification.

We first confirm the presence of a cointegration relationship with possible breaks by a test of Gregory and Hansen [9]. Then, exploiting the fact that short-run nominal interest rates had declined almost monotonically since the early 1990s (see Figure 1), sophisticated tests of structural breaks for interest-rate semi-elasticity of money demand, constructed by Hansen [11] and Kuo [14] allow us to identify the rate of interest below which money demand is extremely interest-elastic. It is important here that econometric tests of structural breaks are used as tests of non-linearity of money demand, not tests of its instability; the latter important issue was raised originally by Lucas [15].

In addition, this paper examines one possible empirical implication of interest-elastic money demand for the quantity theory of money. As pointed out above, with extremely interest-elastic money demand, it may be hard for the quantity theory to hold fully. This paper tests the extent to which nominal prices respond to changes in money supply in the short-run before and during the period of interest-elastic money demand.

This paper is organized as follows. Section 2 discusses an empirical specification and its implication for the quantity theory of money. Based on the two estimation methods, the dynamic ordinary least squares (OLS) estimator and the fully modified OLS estimator, Section 3 demonstrates empirically that extremely highly interest-elastic money demand curves emerged under the low interest rate policy, and that it substantially weakened the relationship between money supply and nominal prices. Finally, Section 4 summarizes our empirical findings. The appendix presents small sample properties of the estimation methods used in the text.

2. Interest-elastic money demand and its implication

Suppose that the demand for real money balances is characterized as a function of real aggregate output and the nominal interest rate in the manner of Cagan [5] or in terms of the following semi-log specification:

$$m_t - p_t = \theta y_t + \frac{1}{\gamma} i_t, \tag{1}$$

where m_t is the logarithm of the nominal money stock at time t, p_t is the logarithm of nominal prices, and i_t is the nominal interest rate. The two parameters θ and γ denote income elasticity and inverse interest-rate semi-elasticity respectively. In the above specification, the closer the absolute value of γ is to zero, the higher the degree of interest-rate semi-elasticity.

The nominal interest rate, assumed to be determined by the Fisher equation, is equal to the sum of the real interest rate r_t and the expected inflation $p^e_{t+1} - p_t$ where p^e_{t+1} denotes the expected future price. For the moment and for simplicity, it is further assumed that $r_t = 0$ and $y_t = 0$. Then, equation (1) reduces to the following rational expectations model:

$$p_t = \frac{1}{1-\gamma} p_{t+1} + \frac{-\gamma}{1-\gamma} m_t$$

The interest-rate semi-elasticity γ has an important positive implication for the properties of the equilibrium path of p_t. In the standard case where the real money balance is a decreasing function of the nominal interest rate ($\gamma < 0$), we obtain the following forward-looking path:

$$p_t = -\gamma \sum_{\tau=0}^{\infty} \left[\left(\frac{1}{1-\gamma} \right)^{\tau+1} m_{t+\tau} \right]. \tag{2}$$

With the above path, the current nominal price reflects both the current and future money supply, and nominal prices respond flexibly to changes in money supply. If money supply increases permanently by an amount Δ_m, then nominal prices rise by the same magnitude. In the case of a permanent change in money supply, therefore, there is a one-to-one correspondence between money supply and nominal prices; that is, the standard quantity theory holds tightly.

Nominal prices are, however, less responsive to non-permanent changes in money supply, as γ is closer to zero and money demand is more interest-elastic. According to the coefficient of the future money supply in equation (2), $-\gamma\left(\frac{1}{1-\gamma}\right)^{\gamma-1}$, as γ is closer to zero, less weight is put on the current and immediate future money supply, and more on the distant future money supply. Therefore, transitory changes in money supply are not significantly reflected in current nominal prices when money demand is extremely interest-elastic.

On the other hand, if the inverse interest-rate semi-elasticity (γ) is positive and smaller than two,[1] then the above forward-looking path is no longer applicable, and the nominal price is determined backward according to the weighted average of p_t and m_t, or $p_{t+1}=(1-\gamma)p_t+\gamma m_t$. In this case, the current nominal price is adjusted gradually from its

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1) The case of positive interest-rate semi-elasticity corresponds to the monetary model with exchange externality proposed by Farmer [7]. The essence of the monetary model presented by Farmer [7] may be expressed with the following money demand function:

$$m_t-p_t=\frac{1}{\gamma}\,i_t+\phi(\bar{m}_t-p_t),$$

where $\bar{m}_t$ equals the amount of nominal money circulating in a macroeconomy. The second term on the right hand side represents exchange externality or the positive effect of the real money balance on transactions: the larger is ϕ, the stronger is exchange externality. Because $m_t=\bar{m}_t$ at equilibrium, the coefficient on the nominal interest rate is equal to $\frac{1}{\gamma}\frac{1}{1-\phi}$ in the reduced form. In this case, the interest-rate semi-elasticity of the reduced form may be positive when exchange externality is strong enough. More concretely, even if γ is negative, the reduced-form interest elasticity is positive under the condition that $\phi>1-\frac{1}{2\gamma}$. Beaudry and Devereux [4] construct a similar monetary model in the two-country setup, thereby exploring implications of a slow adjustment for real exchange rates.

lagged level in response to only the current money supply. On the border where the interest-rate semi-elasticity is infinite or $\gamma = 0$, nominal interest rates have to be zero. Then, we obtain

$$p_{t+1} = p_t$$

under the assumption that $r_t = 0$. With an infinite interest-rate semi-elasticity, nominal prices do not respond at all to either current or future money supply. Even allowing for time-varying real interest rates r_t and real output y_t, we still obtain the above borderline property, or

$$p_{t+1} = p_t - \gamma_t.$$

The above discussion suggests the following positive implication for the quantity theory of money. When γ is negative, but close to zero, current nominal prices may not be sensitive to changes in money supply in two cases. First, if a central bank cannot make a firm commitment to permanent increases in money supply, policy shocks on money supply turn out to be transitory; therefore, as discussed above, current nominal prices do not respond to transitory changes in money supply when negative γ is close to zero. Second, when market participants are myopic and consider only current and immediate future money supply, we have the same implication for a money-price relationship as in the first case.

As is well known, interest-elastic money demand has one important normative implication as well. When γ is negative, but close to zero, a marginal increase in nominal interest rates involves larger deadweight losses, or greater money-holding costs. In other words, lowering interest rates under interest-elastic money demand would involve large welfare costs in the future when an interest rate is raised again. As the preceding discussion demonstrates, a degree of interest-rate semi-elasticity $\frac{1}{\gamma}$ has important positive and normative implications.

3. Specifications and estimation results

3.1. Methods and data

In this section, we estimate empirically the shape of the Japanese money demand function with due consideration of the possibility that interestrate semi-elasticity is extremely large under the low interest rate policy. More concretely, as Figure 3 describes, we approximate money demand by a combination of two linear functions with different degrees of semi-elasticity (that is, small semi-elasticity for high and middle interest rates and large semi-elasticity for low interest rates). Exploiting the fact that short-run nominal interest rates had declined almost monotonically since the early 1990s, tests for structural breaks with respect to semi-elasticity allows us to identify the nominal interest rate below which extremely interest-elastic money demand emerges.

For example, if a semi-elasticity structural break is detected when the BOJ started to implement the low interest rate policy in 1995, then it may be possible to characterize money demand with a combination of two different degrees of interest-rate semi-elasticity. As discussed in the introduction, therefore, it is important in our context that we use tests of structural breaks not as a test of instability, but as a test of non-linearity. In addition, we examine one empirical implication of interest-elastic money demand by estimating shortrun responses of nominal prices to changes in money supply. As suggested in the previous section, it is expected that current nominal prices may not be sensitive to changes in money supply under the low interest rate regime.

For estimation, we specify a money demand equation as follows:

$$m_t - p_t = \text{constant} + \alpha y_t + \beta i_t + \varepsilon_t, \tag{3}$$

Figure 3: Highly Interest-elastic Money Demand in the Neighborhood of Zero Nominal Interest Rate

where α implies income elasticity, and β denotes interest-rate semi-elasticity. The last term ε_t represents a stochastic shock to money demand.

Employing a method proposed by Gregory and Hansen [9], we first test the absence of a cointegration relationship in equation (3) against the presence of cointegration with possible structural breaks. While, as Gregory and Hansen [9] emphasized, their test is powerful for the purpose of rejecting the absence of cointegration, it cannot identify points of structural breaks for the following reasons. First, this test includes cointegration without any regime shift as a special case of the alternative hypothesis. Second, the ordinary least squares (OLS) estimation on which this test relies is not efficient, and does not generally have an asymptotic standard distribution under the hypothesis of cointegration.

Hence, once we have rejected the absence of cointegration for equation (3) using Gregory and Hansen [9], we use the tests proposed by

Hansen [11] and Kuo [14] to identify points of structural breaks. Their methods allow us to test for cointegration without any break against cointegration with breaks. For their tests, the OLS estimator is not appropriate. Considering the contemporaneous and intertemporal correlation between changes in the explanatory variables (y_t and i_t) and the disturbance in money demand (ε_t), we estimate equation (3) by the dynamic OLS estimator proposed by Saikkonen [22], Stock and Watson [23], and others, and the fully modified OLS estimator proposed by Phillips and Hansen [20]. The main purpose of adopting the two estimation methods is to carefully examine the robustness of the test and estimation results.

To test for the presence of structural breaks under cointegration, we choose the test proposed by Hansen [11] for a pure structural change where constancy in the whole set of parameters is tested against parameter instability. The test proposed by Kuo [14] is designed to test for a partial structural change where constancy in subsets of parameters is examined. In both tests of structural changes, the null hypothesis of cointegration with parameter stability is tested against the alternative hypothesis of cointegration with parameter instability.[2)]

As Hansen [11] suggests,[3)] the asymptotic distribution for tests of structural changes, derived by Hansen [11] and Kuo [14], is applicable not only to the fully modified OLS, but also to the dynamic OLS. The estimation results based on the dynamic OLS are first presented, and are then carefully compared with those based on the fully modified

2) Although our investigation of structural breaks takes cointegration with parameter constancy as the null hypothesis using test statistics based on the limiting distribution, alternative methods have been explored in depth in the econometric literature on structural breaks. For example, Andrews, Lee, and Ploberger [1] examine the finite sample property of a test of cointegration with structural breaks. Our appendix reports the small sample properties of our estimation methods.

3) Hansen [11] (p.332) mentions that, because the fully modified OLS and the dynamic OLS are asymptotically equivalent, the test statistics would have the same asymptotic distributions as those tabulated in his article.

Table 1: Unit Root Tests on $m_t - p_t$, y_t, and i_t
1985:8-2001:3

	Test Statistics			
	$ADF-t$	$ADF-Z$	$PP-Z_t$	$PP-Z_a$
real M1	0.65	0.57	0.82	0.54
real M2+CD	−2.53	−3.23	−3.75**	−2.60
real Currency	−0.50	−0.56	−1.18	−0.72
y_t	−2.50	−8.61	−2.19	−6.07
i_t	−1.07	−1.61	−1.08	−2.25
Δ(real M1)	−3.39*	−46.83**	−13.31**	−228.3**
Δ(real M2+CD)	−2.15	−10.68	−11.5	−219.1**
Δ(real Currency)	−6.35**	−106.6**	−15.32**	−288.5**
Δy_t	−3.97**	−50.34**	−19.07**	−343.5**
Δi_t	−4.15**	−38.49**	−11.74**	−197.0**

1 . The $ADF-t$ and $ADF-Z$ indicate the augmented Dickey Fuller t and Z statistics, respectively (Dickey and Fuller, 1979).
2 . The $PP-Z_t$ and $PP-Z_a$ indicate the Phillips-Perron Z_t and Z_a statistics, respectively (Phillips and Perron, 1988).
3 . y_t is specified as an $I(1)$ with drift, while i_t is specified as an $I(1)$ without drift.
4 . *and **indicate the 5% and 1% levels of significance, respectively.

OLS.

For our estimation, the sample period is January 1985 to March 2001. The principal reason for excluding the period before 1985 is that Japanese money markets had been strictly regulated until the mid-1980s. It is only since the mid-1980s that commercial banks and securities companies have been allowed to issue various kinds of money market instruments at market rates. Therefore, money market rates were unlikely to have properly reflected market conditions before 1985. As suggested in the introduction, a major reason for omitting the period after March 2001 is that a degree of interest-rate semi-elasticity is not identifiable at all for a flat money demand (with infinite semi-elasticity) at zero interest rates caused by the quantity easing policy from March 2001.

We build the set of monthly data as follows. As nominal monetary

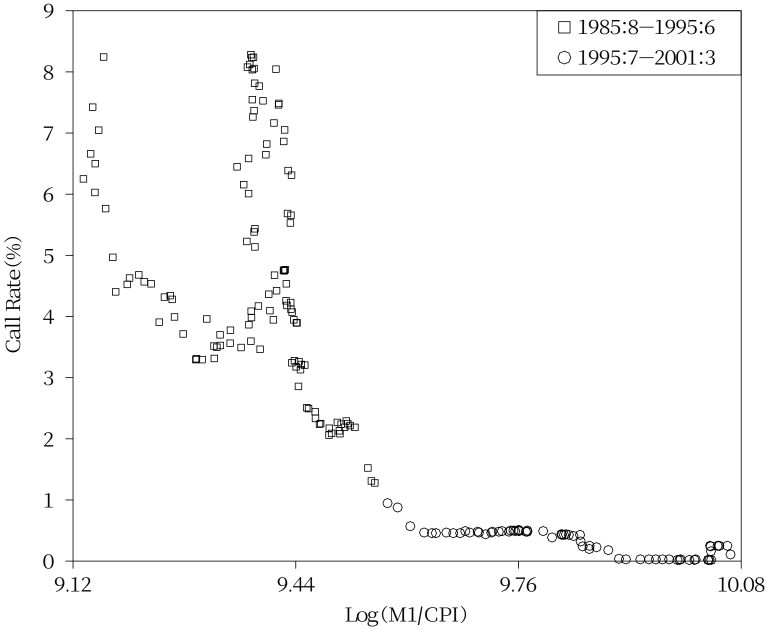

Figure 4: Call Rates and Real Monetary Balance

aggregates we choose M1, M2+CD, and currency. These data are compiled by the BOJ. Of these three monetary aggregates, we are particularly interested in the estimation results based on M1 because M1 reflects to a greater extent the transaction demand for money than do the other two aggregates.

The consumer price index constructed by the Statistics Bureau is used for nominal prices, and the industrial production index documented by the Ministry of International Trade and Industry is adopted for real aggregate output. Overnight call rates, reported by the BOJ, are used as nominal interest rates. All data are recorded in monthly averages. As for both nominal monetary aggregates and industrial production, our data set is based on variables that are officially seasonally adjusted by the above reporting agencies. The con-

Table 2: Residual Based Tests for Cointegration with Regime Shifts 1985:8-2001:3

	M1	M2+CD	Currency	5% c.v.
Inf-ADF	−4.50	−3.20	−4.13	−5.50
	(1998:1)	(1995:6)	(1998:9)	
Inf-Z_t	−6.11**	−4.38	−5.78**	−5.50
	(1998:1)	(1995:6)	(1998:1)	
Inf-Z_α	−61.81**	−32.44	−54.63*	−58.33
	(1998:1)	(1995:6)	(1998:1)	

1 . Tests are based on the regime shift model proposed by Gregory and Hansen (1996).
2 . Critical values refer to Gregory and Hansen (1996).
3 . Data points with the lowest ADF, Z_t and Z_α statistics are reported in parentheses.
4 . In the case of Inf-ADF, the lag length is selected on the basis of the t-test constructed by Gregory and Hansen (1996).
5 . *and **indicate the 10% and 5% levels of significance, respectively.

sumer price index is seasonally adjusted by the X11 method based on the sample period 1970 to 2001.

As shown in Table 1, unit root tests for the real money balance, real output, and nominal interest rates (call rates) fail to reject the presence of unit roots for levels, and succeed in rejecting unit roots for first differences in most cases. These results suggest that the processes of those variables are integrated of order 1. As shown by Figure 4, the real money balance (M1) seemed much more elastic with respect to call rates from mid-1995.

In the main text, the estimation results based on asymptotic distributions are reported, while those based on small samples are described in the appendix.

3.2. Tests of no cointegration against cointegration with breaks

This subsection reports the Gregory and Hansen [9] test results. According to Inf-Z_t and Inf-Z_α statistics reported in Table 2, the null hypothesis (no cointegration) is rejected strongly for both M1 and Currency. In the case of M2+CD, however, both statistics fail to reject no cointegration. This result is consistent with Miyao [16], who also rejects the presence of cointegrating M2 demand functions using Japanese money market data. Based on the latter result, we do not apply Hansen [11] or Kuo [14] tests to M2+CD.[4)]

Small sample properties of Gregory and Hansen [9] are reported in the appendix. As the appendix demonstrates, the estimates used in this study are not subject to serious small sample problems.

3.3. Tests of structural breaks

We employ the LM (Lagrange multiplier) test using either the dynamic OLS or the fully modified OLS estimation for tests of cointegration with parameter stability against pure or partial structural changes. The first step in the test procedure for a pure structural change is to choose a break point T, and construct a set of time-varying parameters (α_t, β_t, constant$_t$) for equation (3) as follows:

$$\text{if } t<T, \text{ then } (\alpha_t,\ \beta_t,\ \text{constant}_t)=(\alpha^1,\ \beta^1,\ \text{constant}^1), \text{ and}$$
$$\text{if } t\geq T, \text{ then } (\alpha_t,\ \beta_t,\ \text{constant}_t)=(\alpha^2,\ \beta^2,\ \text{constant}^2).$$

Next, we compute the LM test statistics to test whether (α^1, β^1, constant^1)=(α^2, β^2, constant^2) based on the estimation results of either the dynamic or fully modified OLS. The resulting LM test statistics are conventionally called F statistics.

Then, the above F statistics are computed for all data points of the sample period. There are two types of tests based on these computed F statistics. When the timing of a structural break is treated as unknown, it is possible to adopt the Sup F test based on the highest F statistic. On the other hand, when the parameters (α_t, β_t, constant$_t$) fol-

4) Although, as discussed above, the Gregory and Hansen [9] test is not appropriate in identifying structural breaks, Table 2 reports these break points estimated by their method. These points are rather different from those documented in Tables 3 and 5.

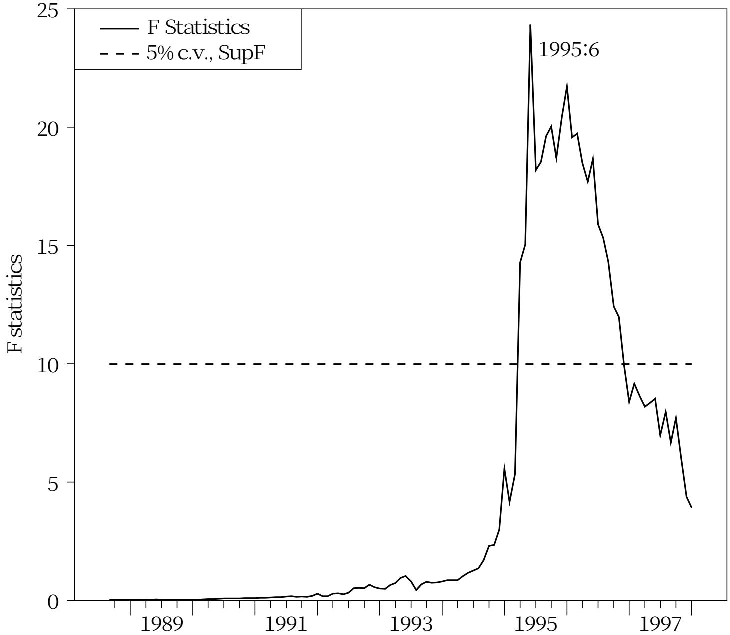

Figure 5: Testing Structural Breaks (M1, Dynamic OLS with k=3)

low a martingale process under the alternative hypothesis,[5)] it is possible to use the Mean F test based on the average F statistic. For a partial structural change, the above procedure is applied to a subset of (α_t, β_t, constant$_t$). We consider a partial structural change to be constancy of either the intercept, income elasticity (α), or interest-rate semi-elasticity (β).

Critical values based on the limiting distribution are available from Hansen [11] for a pure structural change, and from Kuo [14] for a partial structural change. As mentioned before, these critical values

5) More concretely, the alternative hypothesis is as follows:

$$(\alpha_{t+1},\ \beta_{t+1},\ \text{constant}_{t+1}) = (\alpha_t,\ \beta_t,\ \text{constant}_t) + (\varepsilon^1_{t+1},\ \varepsilon^2_{t+1},\ \varepsilon^3_{t+1})$$

where $E_t\varepsilon^1_{t+1} = E_t\varepsilon^2_{t+1} = E_t\varepsilon^3_{t+1} = 0$.

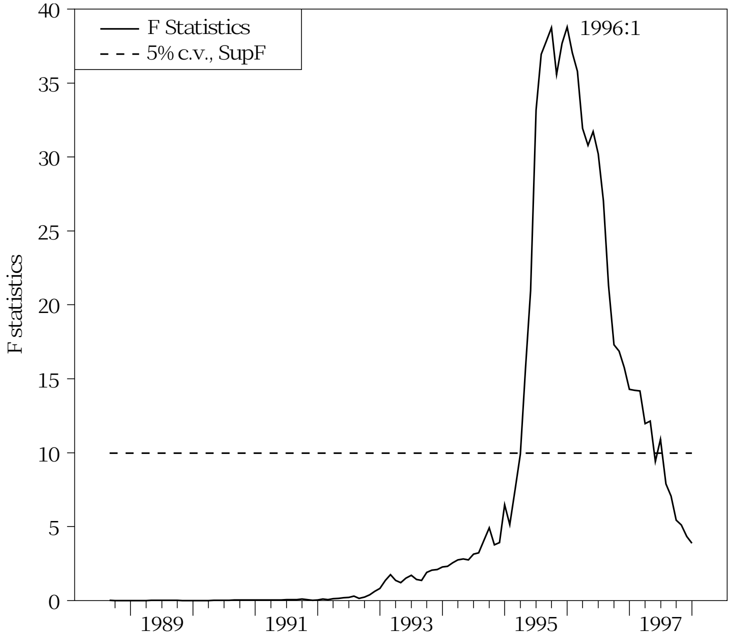

Figure 6: Testing Structural Breaks (M1, Dynamic OLS with k=6)

are applicable to the dynamic OLS and the fully modified OLS. In the case of the dynamic OLS, it is necessary to choose the number of leads and lags, denoted by k. We set k to be either three or six, following preceding studies including those of Stock and Watson [23]. While $k=3$ is most preferred for our data set according to the Schwarz information criterion, the estimation results do not depend on the choice of k.

As an example of the above procedure, Figure 5 plots the F statistic for each data point, together with the 5% critical value of the Sup F test for the case of constancy of β based on the dynamic OLS estimation with $k=3$. As this figure clearly shows, the highest F statistic at June 1995 far exceeds the 5% critical values of the Sup F test. This result therefore implies that constancy of interest-rate semi-elasticity is strongly rejected given an unknown break point. Similarly, Figure 6 demonstrates that the F statistic is highest at January 1996 in the case of the dynamic OLS estimation with $k=6$.

The appendix reports the small sample properties of tests proposed by Hansen [11] and Kuo [14]. As the appendix demonstrates, the estimates used in this study are not subject to serious small sample problems.

Estimation results based on the dynamic OLS The upper panel of Table 3 summarizes the test results for parameter stability for the dynamic OLS with $k=3$. In terms of pure structural changes, the Sup F rejects parameter stability for M1 at the 5% level of significance and Currency at the 10%. Instability of interest-rate semi-elasticity is mainly responsible for these results for pure structural changes. As shown by the two rows denoted by (4), constancy of interest-rate semi-elasticity is rejected for M1 at the 5% level of significance by both the Sup F test and the Mean F test. For the instability of interest-rate semi-elasticity, the F statistic is highest at June 1995 for the two monetary aggregates.

The lower panel of Table 3, on the other hand, reports the test results for the dynamic OLS with $k=6$. The overall results are fairly similar to those of $k=3$. In the case of both pure structural changes and instability of interest-rate semi-elasticity, the F statistic is highest at January 1996 for M1, and September 1995 for Currency.

As Hansen [11] emphasizes, it would be inappropriate to conclude, based on the rejection of the Sup F test, that there are two cointegrating regimes separated by the data point with the highest F statistic. This is particularly so where there is no prior knowledge of the break points. In other words, the rejection result would allow for various kinds of alternative hypotheses. Before the empirical investigation, however, we had the legitimate expectation that a structural break would occur around 1995 when the BOJ implemented the low interest rate policy. Given this expectation, one of the most natural

Table 3: Tests for Parameter Instability of Money Demand Equations by Dynamic OLS

k=3	M1	Currency	5% c.v.
Sup F			
(1)	26.46**	13.63*	17.3
	(1995:6)	(1995:6)	
(2)	1.04	3.72	10.75
	(1995:4)	(1995:6)	
(3)	0.80	3.45	10.71
	(1995:4)	(1995:6)	
(4)	24.33**	12.05**	9.98
	(1995:6)	(1995:6)	
Mean F			
(1)	5.96	3.43	7.69
(2)	0.24	0.80	2.22
(3)	0.18	0.73	2.14
(4)	4.49**	2.22*	2.47

k=6	M1	Currency	5% c.v.
Sup F			
(1)	41.90**	26.57**	17.3
	(1996:1)	(1995:9)	
(2)	1.80	4.08	10.75
	(1996:6)	(1996:8)	
(3)	9.13	3.41	10.71
	(1996:7)	(1997:5)	
(4)	38.79**	24.83**	9.98
	(1996:1)	(1995:9)	
Mean F			
(1)	8.67**	7.28*	7.69
(2)	0.42	1.13	2.22
(3)	1.49	0.72	2.14
(4)	7.09**	5.31**	2.47

1. Tests are based on the dynamic OLS proposed by Stock and Watson (1993) with the number of leads and lags equal to either three or six.
2. Critical values refer to Kuo (1998) for a partial structural change, and to Hansen (1992) for a pure structural change.
3. In each panel, the first row denoted by (1) refers to testing the whole cointegrating vector, (2) to testing the intercept, (3) to testing the coefficient on y_t, and (4) to testing the coefficient on i_t.
4. Data points with the highest F statistics are reported in parentheses.
5. *and **indicate the 10% and 5% levels of significance, respectively.

Table 4: Parameter Estimates of Money Demand Equations by Dynamic OLS

No. of Leads and Lags	m_t	Point Estimates (Standard Errors)		
		constant	α	β
k=3	M1			
	1985:8-2001:3	2.716(9.604)	1.538(2.081)	−0.072(0.050)
	1985:8-1995:5	4.010(10.59)	1.214(2.331)	−0.041(0.085)
	1995:6-2001:3	2.958(1.776)	1.535(0.385)	−0.690(0.054)
	Currency			
	1985:8-2001:3	−0.044(3.848)	1.833(0.834)	−0.057(0.020)
	1985:8-1995:5	0.056(3.976)	1.791(0.875)	−0.040(0.032)
	1995:6-2001:3	3.715(2.901)	1.055(0.630)	−0.523(0.088)
k=6	M1			
	1985:8-2001:3	2.704(14.38)	1.540(3.112)	−0.072(0.062)
	1985:8-1995:12	3.656(14.47)	1.292(3.135)	−0.038(0.089)
	1996:1-2001:3	5.924(2.000)	0.896(0.433)	−0.690(0.029)
	Currency			
	1985:8-2001:3	−0.058(5.740)	1.836(1.241)	−0.057(0.025)
	1985:8-1995:8	0.142(8.259)	1.772(1.794)	−0.040(0.052)
	1995:9-2001:3	1.924(4.229)	1.445(0.917)	−0.565(0.073)

1. The estimation method is based on the dynamic OLS proposed by Stock and Watson (1993) with the number of leads and lags equal to either three or six.
2. Standard errors in parentheses are based on AR(3) spectral estimators for k=3, and AR(6) spectral estimators for k=6.

possibilities would be that a structural break occurred at the data point with the highest F statistic, given that the Sup F test indicates that a break point is close enough to 1995. We pursue this possibility in the following.

We assume that there are two cointegrating regimes separated by the data point with the highest F statistic: June 1995 for k= 3 and either January 1996 or September 1995 for k= 6. Table 4 reports the parameters of money demand functions estimated by the dynamic OLS for each cointegration regime. There are some interesting observations on the parameter estimates. First, money demand was estimated to be much more elastic with respect to nominal interest rates in the second period than in the first period. In particular, the absolute value of interest-rate semi-elasticity of M1 shows a remarkable increase. The estimated β changes from -0.041 to -0.690 for $k=3$, and from -0.038 to -0.690 for $k=6$.

Second, compared with existing empirical results on M1 demand functions, money demand is remarkably interest-elastic in the above second sub-sample. For Japan, Miyao [17] reports that interest-rate semi-elasticity is -0.07 for a sample period 1980 to 1996. For the US, on the other hand, Stock and Watson [23] apply several estimation methods for a sample period 1946 to 1987, and find the estimated interest-rate semi-elasticity ranges from -0.02 to -0.09 excluding cases with positive elasticity. By extending the postwar US data through 1996, Ball [3] demonstrates that interest-rate semi-elasticity is around -0.05. Among existing empirical studies of M1 demand, we cannot find any point estimates of interest-rate semi-elasticity around -0.7.[6)]

Third, income elasticity estimates are often imprecise, exhibiting large standard errors. In any sample period, full or sub, estimated α for M1 demand functions is not significantly different from unity. This result for income elasticity contrasts with that based on the fully modified OLS. As reported in Table 6, the fully modified-OLS-estimated α has much smaller standard errors, and is closer to unity for the first sub-sample, while it is not statistically different from zero for the second sub-sample.

Considering that the second sub-sample period is rather short, we conduct a bootstrap estimation, and report estimation results in the appendix. As the appendix demonstrates, the estimation results do not change substantially even if short sample periods are considered ex-

6) Hoffman, Rasche, and Tieslau [12] estimate M1 demand for five industrial countries including the US, Japan, Canada, the UK, and West Germany. However, they estimate interest elasticity, not interest semi-elasticity, and thus it is impossible to compare our results directly with theirs.

Table 5: Tests for Parameter Instability of Money Demand Equations by Fully Modified OLS

	M1	Currency	5% c.v.
Sup F			
(1)	18.27**	33.65**	17.3
	(1995:7)	(1994:12)	
(2)	6.85	20.17**	10.75
	(1991:4)	(1989:2)	
(3)	6.87	20.18**	10.71
	(1991:4)	(1989:2)	
(4)	11.31**	27.26**	9.98
	(1995:5)	(1994:12)	
Mean F			
(1)	7.48*	14.69**	7.69
(2)	1.81*	2.63**	2.22
(3)	1.81*	2.59**	2.14
(4)	4.79**	3.32**	2.47

1. Tests are based on the fully modified OLS proposed by Hansen (1992).
2. Critical values refer to Kuo (1998) for a partial structural change, and to Hansen (1992) for a pure structural change.
3. In each panel, the first row denoted by (1) refers to testing the whole cointegrating vector; (2) to testing the intercept, (3) to testing the coefficient on y_t, (4) to testing the coefficient on i_t.
4. Data points with the highest F statistics are reported in parentheses.
5. *and **indicate the 10% and 5% levels of significance, respectively.

plicitly.

Estimation results based on the fully modified OLS We now report the test and estimation results based on the fully modified OLS.[7] We carefully compare these with the above results based on the dynamic OLS. Table 5 reports the parameter instability test results. As shown in the two rows denoted by (1), both the Sup F and the Mean F tests indicate that there are significant pure structural changes for

7) We estimate money demand equations by the fully modified OLS using the GAUSS code programmed by Professor Bruce Hansen. In his estimation procedure, the quadratic spectrum kernel is chosen, and the selection of an optimal bandwidth is based on Andrews and Monahan [2].

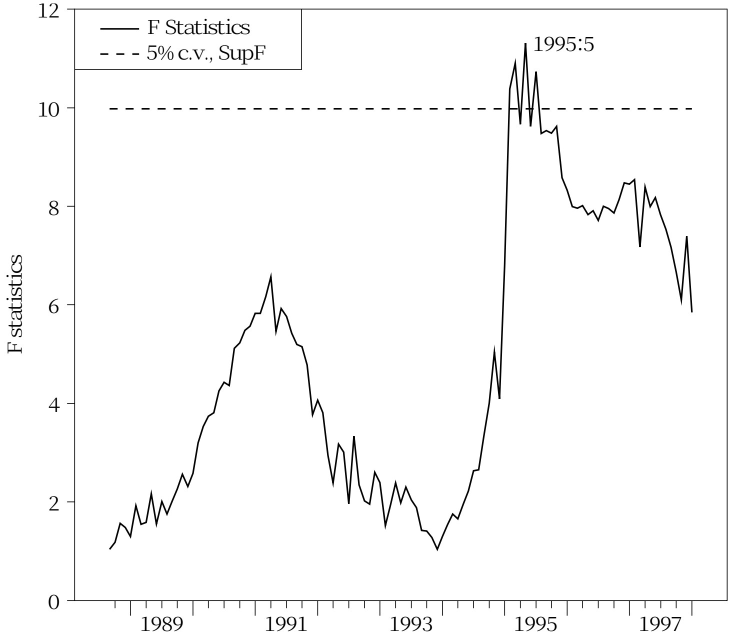

Figure 7: Testing Structural Breaks (M1, Fully Modified OLS)

the two monetary aggregates. More concretely, the data point with the highest F statistic is July 1995 for M1, and December 1994 for Currency. In terms of M1 demand functions, the fully modified OLS indicates that the break point occurred one month after what is implied by the dynamic OLS with k=3.

According to the partial structural change test, the instability of interest-rate semielasticity is most responsible for a pure structural change among the intercept, α, and β. Figure 7 plots the F statistic for each data point, together with the 5% critical value of the Sup F test for the case of constancy of interest-rate semi-elasticity (β). As this figure shows, the F statistic is highest at May 1995.

Table 6 reports parameter estimates of money demand functions from the fully modified OLS by assuming that two cointegrating reg-

Table 6: Parameter Estimates of Money Demand Equations by Fully Modified OLS

	constant	α	β
M1			
1985: 8 -2001: 3	0.510(6.675)	2.022(1.450)	−0.077(0.040)
1985: 8 -1995: 6	4.114(0.228)	1.191(0.050)	−0.036(0.002)
1995: 7 -2001: 3	7.958(1.678)	0.447(0.363)	−0.571(0.059)
Currency			
1985: 8 -2001: 3	6.450(2.385)	0.416(0.518)	−0.058(0.014)
1985: 8 -1994:11	0.448(0.666)	1.711(0.147)	−0.047(0.006)
1994:12-2001: 3	6.163(1.717)	0.525(0.371)	−0.499(0.060)

1 . The estimation method is based on the fully modified OLS proposed by Hansen (1992).
2 . Standard errors are in parentheses.

imes are separated by the data point with the highest F statistic for pure structural changes. There are two differences in the estimation results between the fully modified OLS and the dynamic OLS. First, interest-rate semielasticity for the second sub-sample is a little less negative. In the case of M1, for example, the fully modified OLS estimate is −0.571, while the dynamic OLS estimate with $k=3$ is −0.690. Second, income elasticity for the first sub-sample is estimated with much smaller standard errors. For M1, income elasticity is close to unity in the first sub-sample, while for the second sub-sample it is not statistically different from zero.

As the preceding estimation results suggest, both the dynamic and fully modified OLS indicate non-linearity in money demand functions as well as the presence of highly interestelastic money demand under the low interest rate regime. In both estimation procedures, the implied break point is fairly close to September 1995 when the BOJ started to develop its low interest rate policy.

Table 7 reports estimation results based on alternative methods for two cointegration regimes separated at July 1995. As this table demonstrates, estimation results based on both Park [19] and Johansen [13] are quite similar to that based on the fully modified OLS, particu-

Table 7: Parameter Estimates of M1 Demand Equations Based on Alternative Methods with a Break at July 1995

	Period	Point Estimates (Standard Errors)		
		constant	α	β
Standard OLS	1985:8-2001:3	2.783(0.475)	1.522(0.103)	−0.071(0.002)
	1985:8-1995:6	4.305(0.120)	1.148(0.026)	−0.035(0.001)
	1995:7-2001:3	6.072(1.162)	0.855(0.251)	−0.572(0.038)
Dynamic OLS (k=3)	1985:8-2001:3	2.716(9.604)	1.538(2.081)	−0.072(0.050)
	1985:8-1995:6	4.013(9.771)	1.213(2.149)	−0.037(0.077)
	1995:7-2001:3	3.307(1.803)	1.460(0.391)	−0.694(0.054)
Dynamic OLS (k=6)	1985:8-2001:3	2.704(14.38)	1.540(3.112)	−0.072(0.062)
	1985:8-1995:6	3.853(21.74)	1.247(4.742)	−0.036(0.137)
	1995:7-2001:3	−1.755(2.572)	2.556(0.558)	−0.746(0.045)
Fully Modified OLS	1985:8-2001:3	0.510(6.675)	2.022(1.450)	−0.077(0.040)
	1985:8-1995:6	4.114(0.228)	1.191(0.050)	−0.036(0.002)
	1995:7-2001:3	7.958(1.678)	0.447(0.363)	−0.571(0.059)
CCR	1985:8-2001:3	2.363(5.662)	1.615(1.219)	−0.090(0.037)
	1985:8-1995:6	4.091(0.224)	1.195(0.049)	−0.036(0.002)
	1995:7-2001:3	6.764(1.843)	0.714(0.398)	−0.630(0.056)
JOH(6)	1985:8-2001:3	25.48(3.420)	−3.081(0.735)	−0.024(0.013)
	1985:8-1995:6	3.307(0.254)	1.366(0.056)	−0.038(0.002)
	1995:7-2001:3	8.738(1.065)	0.309(0.230)	−0.720(0.031)

1. CCR is the canonical cointegrating regression estimator proposed by Park (1992).
2. JOH(k) is the maximum likelihood estimator proposed by Johansen (1991) and evaluated using k lagged first differences.

larly for estimates of α.

3.4. Short-run responses to changes in money supply

The previous empirical results strongly suggest that money demand was extremely interest-elastic. As discussed in Section 2, highly interest-elastic demand may make nominal prices unresponsive to changes in money supply when a central bank cannot make a firm commitment to permanent changes in money or market participants do not have long-run expectations about money supply. In this subsection, we empirically examine whether such a phenomenon indeed emerged as a result of the highly interest-elastic money demand we

Table 8-1: Parameter Estimates of Error Correction Type Models Based on Dynamic OLS (k=3)

m_t	Lags	γ_0^m	γ_1^m	γ_0^y	γ_1^y	μ_0	μ_1	F
M1	1	0.259	0.028	−0.152	−0.068	0.115	−0.004	7.575
		(0.097)	(0.065)	(0.405)	(0.095)	(0.222)	(0.030)	(0.006)
	2	0.239	0.025	−0.033	−0.059	0.044	−0.007	8.203
		(0.074)	(0.053)	(0.140)	(0.055)	(0.072)	(0.013)	(0.004)
	3	0.169	0.020	0.073	−0.053	−0.023	−0.007	4.949
		(0.059)	(0.051)	(0.056)	(0.050)	(0.029)	(0.010)	(0.027)
	4	0.152	0.015	0.032	−0.046	0.012	−0.009	5.055
		(0.052)	(0.046)	(0.040)	(0.043)	(0.022)	(0.009)	(0.025)
Currency	1	0.142	−0.033	0.136	0.094	−0.033	−0.056	1.873
		(0.104)	(0.163)	(0.142)	(0.347)	(0.064)	(0.112)	(0.172)
	2	0.143	0.018	0.117	−0.047	−0.023	−0.010	2.236
		(0.076)	(0.071)	(0.102)	(0.060)	(0.045)	(0.013)	(0.136)
	3	0.129	0.016	0.079	−0.069	−0.013	−0.011	2.316
		(0.054)	(0.065)	(0.050)	(0.051)	(0.022)	(0.011)	(0.129)
	4	0.148	0.011	0.048	−0.054	−0.000	−0.015	3.776
		(0.049)	(0.063)	(0.037)	(0.044)	(0.017)	(0.009)	(0.053)

1. The error correction type model is specified as
$$\Delta p_t = \gamma_0^m I_{year<break} \Delta m_t + \gamma_1^m I_{year \geq break} \Delta m_t$$
$$+ \gamma_0^y I_{year<break} \Delta y_t + \gamma_1^y I_{year \geq break} \Delta y_t + \mu_0 I_{year<break} z_t - 1 + \mu_1 I_{year \geq break} z_t - 1,$$
where z_t is defined as $z_t = (m-p)_t - (\text{constant} + \alpha y_t + \beta i_t)$ using the estimation result of the dynamic OLS with k=3.
2. Instrumental variables include constant, lagged Δm_t, and lagged Δy_t. The number of lags for instrumental variables is controlled from one to four.
3. Standard errors are in parentheses.
4. The last column reports the F statistics of $\gamma_0^m = \gamma_1^m$. P values of the F statistics are in Parentheses.

detected. To differentiate the effect of money supply on nominal prices between the pre-break and post-break periods, we estimate the following equation:

$$\Delta p_t = \text{constant} + \lambda_0^c I_{date<break} + \lambda_0^m I_{date<break} \Delta m_t + \lambda_1^m I_{date \geq break} \Delta m_t$$
$$+ \lambda_0^y I_{date<break} + \Delta y_t + \lambda_1^y I_{date \geq break} \Delta y_t$$
$$+ \mu_0 I_{date<break} \varepsilon_{t-1} + \mu_1 I_{date \geq break} \varepsilon_{t-1} + \xi_t,$$

where I is the indicator function dependent on the condition defined

Table 8-2: Parameter Estimates of Error Correction Type Models Based on Fully Modified OLS

m_t	Lags	γ_0^m	γ_1^m	γ_0^y	γ_1^y	μ_0	μ_1	F
M1	1	0.310	0.025	−0.165	−0.065	0.129	−0.005	4.470
		(0.187)	(0.068)	(0.475)	(0.090)	(0.277)	(0.023)	(0.035)
	2	0.260	0.025	−0.047	−0.060	0.053	−0.006	6.588
		(0.101)	(0.055)	(0.173)	(0.058)	(0.097)	(0.010)	(0.011)
	3	0.164	0.019	0.076	−0.052	−0.029	−0.005	4.490
		(0.060)	(0.051)	(0.056)	(0.049)	(0.032)	(0.008)	(0.035)
	4	0.155	0.013	0.032	−0.040	−0.001	−0.005	5.276
		(0.053)	(0.046)	(0.039)	(0.043)	(0.024)	(0.007)	(0.022)
Currency	1	0.199	−0.054	0.110	0.313	−0.013	−0.131	8.044
		(0.121)	(0.206)	(0.184)	(0.822)	(0.081)	(0.271)	(0.005)
	2	0.191	0.005	0.064	−0.054	−0.001	−0.010	7.290
		(0.050)	(0.064)	(0.066)	(0.057)	(0.026)	(0.012)	(0.007)
	3	0.166	0.002	0.043	−0.080	0.006	−0.012	5.636
		(0.045)	(0.061)	(0.043)	(0.049)	(0.018)	(0.011)	(0.018)
	4	0.168	−0.001	0.030	−0.058	0.011	−0.017	6.248
		(0.046)	(0.060)	(0.035)	(0.043)	(0.015)	(0.010)	(0.013)

1. See the footnotes to Table 8-1.

in subscripts. For example, if a data point is before a break, then $I_{date<break}$ is one, otherwise zero. The final term, ξ_t, represents a stochastic disturbance. In addition, the lagged ε_t defined by the dynamic OLS estimation of equation (3) serves as the error correction term.

If a weak short-run relationship between nominal prices and money supply was created by the low interest rate policy, then we expect $\lambda_0^m > 0$ and $\lambda_1^m = 0$. In addition, we may have $\lambda_0^m \neq \lambda_1^m$. With respect to the coefficients on the error correction terms, we expect $\mu_0 > 0$ and $\mu_1 > 0$ if there is a quick recovery to long-run equilibrium.

The estimation of equation (4) requires instrumental variable estimation to control for simultaneity biases. We include as instrumental variables, constant terms, lagged changes in money supply, and lagged real output increases. The number of lags is controlled from one to four. Table 8-1 reports our estimation results for the dynamic OLS with $k=3$. In this estimation, we set June 1995 as a break for the two monetary aggregates.[8)]

The most important finding is that λ_0^m is significantly positive, whereas λ_1^m is not significantly different from zero. The contrast between λ_0^m and λ_1^m is most remarkable for M1 money demand; $\lambda_0^m=\lambda_1^m$ is rejected statistically as shown by the last column of Table 8-1. On the other hand, both λ_0^y and λ_1^y are insignificant. The coefficients on the error correction terms are also insignificant, suggesting that the path returned to long-run equilibrium quite slowly. Although not reported in this table, the result for $k=6$ is similar to that for $k=3$. As shown in Table 8-2, the estimation results do not change substantially even if break points are based on the fully modified OLS.

Our findings clearly suggest that nominal prices responded immediately, though only partially, to changes in the money supply in the pre-break period, but not at all in the post-break period.

4. Conclusion

Exploiting the fact that nominal interest rates had declined monotonically since the early 1990s, we used a test of structural breaks to investigate whether money demand is non-linear.

We found that money demand is extremely interest-elastic when nominal interest rates are below 0.5 percent per annum. Our findings indicate that interest-elastic money demand emerged even before the BOJ implemented its zero interest rate policy in 1999 and its quantity easing policy in 2001. In addition, consistent with the positive implication of high interest-rate semi-elasticity, we have demonstrated that nominal prices did not respond to changes in money supply in the short-run between 1995 and 2001. While the test results reported in the text are based on asymptotic distributions, the appendix

8) Because the estimated constant term of equation (4), if included, is not significantly different from zero, this table reports the case without any constant term.

shows that they also have robust small sample properties.

We can infer from these results that raising short-run interest rates above 0.5 percent may be a precondition for the quantity theory of money to hold, but that this may force economic agents to bear large welfare losses given extremely interest-elastic money demand.

Appendix: Small sample properties of test statistics and parameter estimates

This appendix examines small sample properties of Hansen [11], and Gregory and Hansen [9] using Monte Carlo methods, and conducts bootstrap estimation for cointegration relationships for the sample period 1995 to 2001.

Monte Carlo studies of a test of Hansen (1992) The Monte Carlo experiment conducted by Gregory, Nason and Watt [10] finds that the power of Hansen's structural break test is considerably poor when the cointegrating error is nearly integrated, and that size distortion (a tendency to reject the null too frequently) is substantial as the number of regressors becomes large, and as the amount of serial correlation of the cointegrating error rises. As reported in Section 3.3, Hansen's tests based on both the dynamic and fully modified OLS indicate the possibility of pure structural changes in money demand functions of the two monetary aggregates. Our test results, however, may be subject to small sample biases as pointed out by Gregory, Nason and Watt [10]. The purpose of this subsection is to investigate the small sample properties of Hansen's tests of structural changes in the context of our sample using Monte Carlo methods.

We calculate the size of Hansen's test for our sample as follows:

1. We estimate a money demand function using a full sample (188 observations) by the dynamic OLS (or the fully modified OLS).
2. A p-th order autoregressive process or AR(p) is estimated for

the residuals of the dynamic OLS (or the fully modified OLS) using the standard OLS. The number of lags is chosen based on the Akaike information criterion.

3. We assume that a pure disturbance in the cointegrating error (ε_t) follows an identically and independently distributed (i.i.d.) normal distribution, and that the standard error of the pure disturbance equals the standard error of the OLS regression of AR(p).
4. We calculate a time series of 188 (the size of our sample) cointegrating residuals in a recursive manner with 188 plus p randomly generated numbers.
5. A time series of real money balances is generated by substituting the resulting residuals as well as the observed explanatory variables y_t and i_t into the money demand function estimated by the dynamic OLS (or the fully modified OLS).
6. We apply Hansen's test for structural changes based on the 5% asymptotic critical values to each set of the sample, and repeat this procedure 5000 times.

Table A-1 reports the rejection frequencies from the above Monte Carlo studies as the size of Hansen's test based on either the dynamic or fully modified OLS. In the case of the dynamic OLS with $k=3$ or $k=6$, neither Sup-F nor Mean-F shows substantial overrejection. The fully modified OLS yields the same result for Sup-F and Mean-F.

We calculate the power of Hansen's test as follows. We first estimate two money demand functions for the first and second halves of the sample period respectively by the dynamic OLS (the fully modified OLS), basing break points on the estimations in Tables 3 and 5. For example, in the case of M1 functions using the dynamic OLS with $k=3$, the first (second) half corresponds to January 1985 to May 1995 (June 1995 to March 2001). Then, we follow the same procedure as in the calculation of size. Table A-1 reports rejection frequencies

Table A-1: Monte Carlo Studies of Small Sample Properties of Tests

		M1	M2+CD	Currency
Hansen's Test (dynamic OLS, k=3)	Size of Test			
	Sup F	0.015	—	0.000
	Mean F	0.000	—	0.000
	Power of Test			
	Sup F	0.699	—	0.687
	Mean F	0.825	—	0.820
Hansen's Test (dynamic OLS, k=6)	Size of Test			
	Sup F	0.003	—	0.037
	Mean F	0.000	—	0.000
	Power of Test			
	Sup F	1.000	—	1.000
	Mean F	0.999	—	0.993
Hansen's Test (fully modified OLS)	Size of Test			
	Sup F	0.025	—	0.005
	Mean F	0.026	—	0.008
	Power of Test			
	Sup F	0.730	—	0.705
	Mean F	0.902	—	0.800
Gregory-Hansen's Test	Size of Test			
	Inf-ADF	0.100	0.103	0.099
	Inf-Z_t	0.077	0.071	0.072
	Inf-Z_α	0.032	0.029	0.032
	Power of Test			
	Inf-ADF	0.093	0.091	0.086
	Inf-Z_t	0.403	0.561	0.465
	Inf-Z_α	0.249	0.374	0.286

1. 5000 simulations are used to calculate rejection rates.
2. Each panel of the size (the power) of tests reports the percentage of times that the statistics are rejected when the critical value is chosen such that the rejection rate for each statistics is 5% under the null (alternative) hypothesis.

under the alternative hypothesis as the power of Hansen's test. As each panel of Table A-1 shows, the calculated power does not deteriorate seriously.

Table A-2: Bootstrap Means and Standard Errors for the Post-Break Sample

m_t	Estimator		constant	α	β
M1	Dynamic OLS (k=3)	Estimates	2.958	1.525	−0.690
			(1.776)	(0.385)	(0.054)
		Bootstrap			
		Means	2.718	1.591	−0.703
		Standard Errors	3.879	0.840	0.140
	Dynamic OLS (k=6)	Estimates	5.924	0.896	−0.690
			(2.000)	(0.433)	(0.029)
		Bootstrap			
		Means	5.613	0.964	−0.694
		Standard Errors	8.005	1.735	0.177
	Fully Modified OLS	Estimates	7.958	0.447	−0.571
			(1.673)	(0.363)	(0.059)
		Bootstrap			
		Means	7.967	0.444	−0.571
		Standard Errors	1.926	0.417	0.072
Currency	Dynamic OLS (k=3)	Estimates	3.715	1.005	−0.523
			(2.901)	(0.630)	(0.088)
		Bootstrap			
		Means	3.515	1.102	−0.535
		Standard Errors	3.435	0.744	0.127
	Dynamic OLS (k=6)	Estimates	1.924	1.445	−0.565
			(4.229)	(0.917)	(0.073)
		Bootstrap			
		Means	1.577	1.522	−0.573
		Standard Errors	7.755	1.681	0.173
	Fully Modified OLS	Estimates	6.163	0.525	−0.499
			(1.717)	(0.371)	(0.060)
		Bootstrap			
		Means	6.149	0.527	−0.497
		Standard Errors	3.252	0.702	0.046

1. Bootstrap estimates are based on 10000 simulations.
2. Standard errors are in parentheses.

These Monte Carlo studies demonstrate that the estimates based on our sample do not suffer from serious small sample biases.

Monte Carlo studies of a test of Gregory and Hansen (1996) To use Monte Carlo studies to test Gregory and Hansen, we assume that a cointegrating error (ε_t) follows a random walk under the null hypothesis, and that ε_t follows an AR(p) process under the alternative hypothesis where no break is assumed. Except for this, we follow the same procedure as for Hansen's test in calculating both size and power. Each panel of Table A-1 reports the calculated size and power of Gregory and Hansen's test. For the size, the Inf-ADF test shows moderate overrejection, while both the Inf-Z_t and Inf-Z_α tests reveal no serious size distortion. As our rejection of the absence of cointegration for M1 and Currency in Section 3.2 is based on the Inf-Z_t and Inf-Z_α tests, our tests are not subject to serious small sample biases. While the power of tests is not our major concern because we reject the null anyway, all the tests indicate poor power.

Bootstrap estimation for the post-break period The sample size for the post-break period may be too small to estimate a cointegration relationship in money demand. This subsection presents bootstrap estimates of money demand functions for the post-break sample period. Bootstrap estimators are based on 10000 simulations. According to Table A-2, while the estimated standard errors are somewhat larger than those based on asymptotic distributions, the sign and significance of estimated parameters do not change substantially.

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